Performance Assessment Task

Which Shape?
Grade 3

This task challenges a student to use knowledge of geometrical attributes (such as angle size, number of angles, number of sides, and parallel sides) to identify and describe shapes. A student must use knowledge of attributes to sort shapes into categories and to distinguish key differences between similar shapes. Student must be able to decompose a geometric figure by drawing lines to subdivide into familiar shapes.

Common Core State Standards Math - Content Standards

Geometry

Reason with shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides) and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
3	2007	9	4	77%

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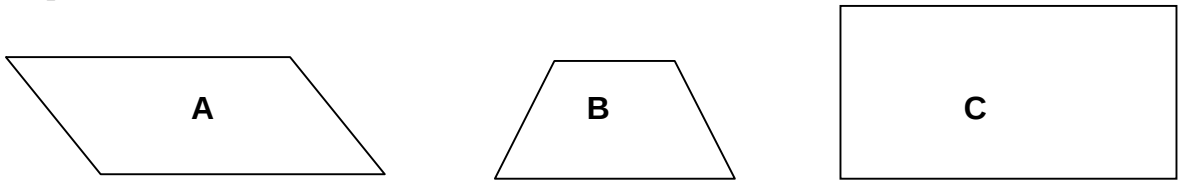
Which Shape?

This problem gives you the chance to:
- identify and describe shapes
- use clues to solve riddles

Use shapes A, B, or C to solve the riddles.



1. I have 4 sides.
   My opposite sides are equal.
   I have 4 right angles.
   Which shape am I? _____

2. I have 4 sides.
   I have only 1 pair of parallel sides.
   Which shape am I? _____

3. I have 4 sides.
   My opposite sides are parallel.
   I do not have any right angles.

   a. Which shape am I? _____

   b. Draw lines that divide my shape into a rectangle and two right triangles.

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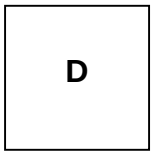
4. Write three different clues for shape D.



Clue 1:

_____

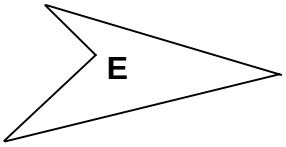
Clue 2:

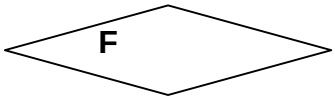
_____

Clue 3:

_____

5. Look closely at shapes E and F.





Write a statement that tells how they are alike.

_____

Write a statement that tells how they are different.

_____

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Task 4: Which Shape?	Rubric	
The core elements of performance required by this task are: • identify and describe shapes • use clues to solve riddles Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **C** or **rectangle**	1	1
2. Gives correct answer: B or **trapezoid**	1	1
3. Gives correct answer: **A** or **parallelogram** Draws lines that are correct.	1 1ft	2
4. Gives three correct statements such as: I have 4 sides. I have 4 right angles. All my sides are equal.	3x1	3
5. Gives two correct statements such as: They both have 4 sides. Shape F has a line of symmetry, and E does not. Shape F has four equal sides, and shape E does not.	2x1	2
Total Points		**9**

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Which Shape?

Work the task. What are some of the attributes that you might expect students to use to describe the shapes in parts 4 and 5? What are some of the attributes or vocabulary that you think will give them difficulty?

Mathematical literacy requires attention to details and constraints that is different from the level of detail needed to make sense of a story. Look at the first part of the task, identifying shapes from clues.

In part one, approximately 11% of the students saw the 4 sides and picked shape A. How many of your students made this choice?

In part two, students saw the parallel sides, but ignored the one pair. 23% picked shape A and 11% picked shape C. How did your students do?

30% of the students saw the no right angles, the last clue read, and picked shape B for part 3.

30% of the students did not attempt to draw lines for part 3b. What types of errors do you see in the student drawings? Which attributes confused students? *(It might be helpful to draw a few examples, so that you can categorize mistakes or think how to set up a discussion about the errors with the class.)*

Look at the clues students used for part 4. List them into categories of correct or incorrect, incomplete. Students often think of mathematical ideas not anticipated by the rubric, so add those to the list as you find them.

Correct Attributes	**Incorrect Attributes**
4 sides	
Equal sides	
4 angles	
Right angles	
Parallel sides	
Line of symmetry	

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How does this help you think about further instruction? What are students confused about? What are their limitations with vocabulary? How can you design activities where students have a reason to use and to practice vocabulary in a meaningful way rather than a memorization of definitions?

Developing the ability to see relevant attributes is related to opportunity and practice. In reading development, students are often given opportunities to compare and contrast. Do students get these same opportunities to be critical thinkers in mathematics?

What are the geometrical attributes that we want students to notice about the shape? Can they see number of sides, size of sides, number and size of angles? Are they thinking about properties like parallel lines and symmetry? How do students think about the concave or reflex angle in shape E? What terms do they use to describe these ideas? Are they talking about points or corners instead of angles? Are some of your students thinking about non-geometrical properties, such as it points to the right or it looks like an arrow?
List some of the incorrect choices that students used to describe the differences.

What are the implications for instruction? Do students have enough opportunities to sort or categorize shapes for themselves? Do they get to make their own labels when sorting? What other activities help students to develop this attention to attributes?

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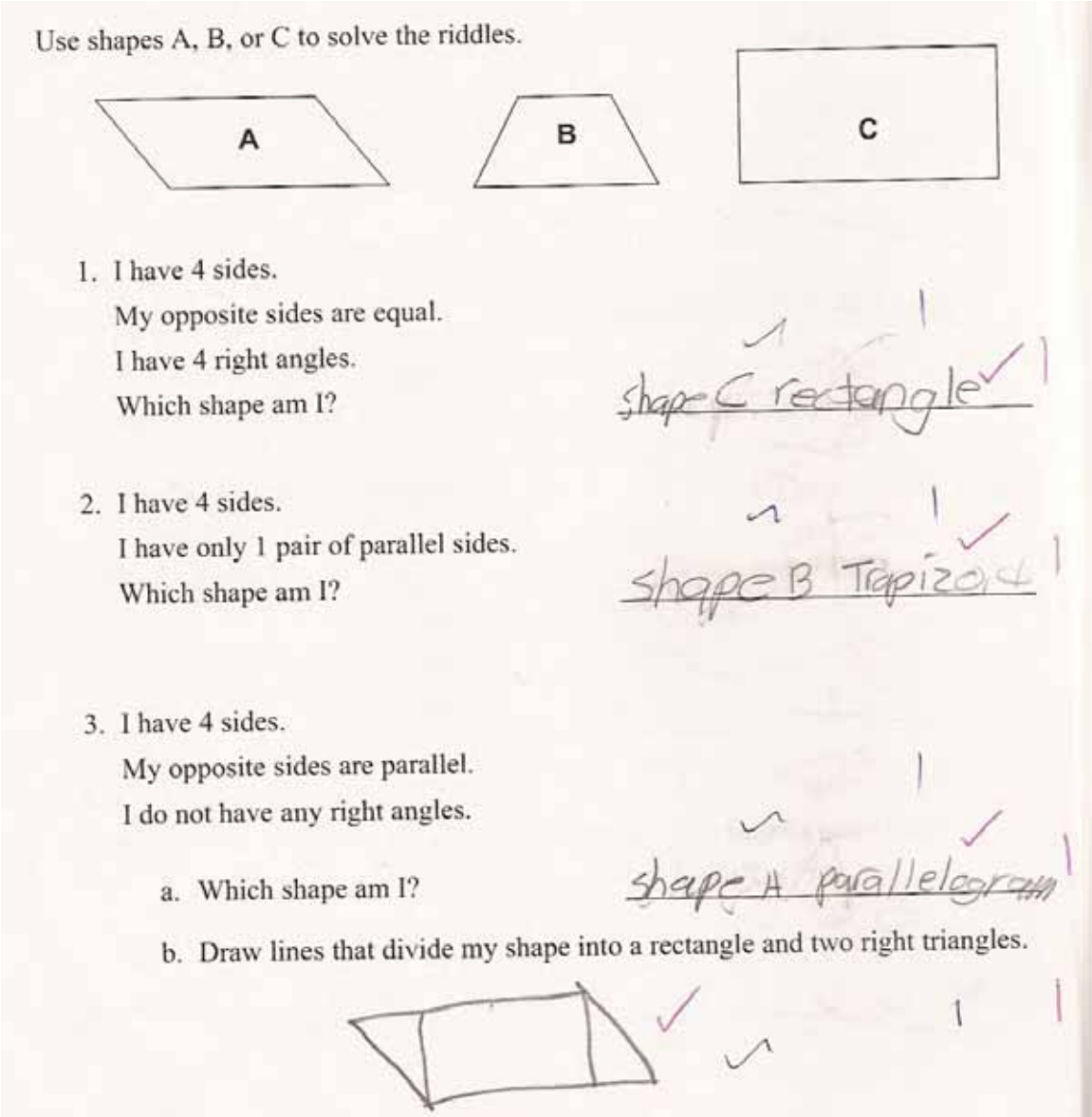
Looking at Student Work on Which Shape?

Student A is able to use the clues to identify the shapes being described and names the shapes. The student draws lines on shape A to meet the constraints or demands of the task. The student uses geometrical vocabulary to describe the attributes in parts 4 and 5.

Student A



Use shapes A, B, or C to solve the riddles.

A B C

1. I have 4 sides.
   My opposite sides are equal.
   I have 4 right angles.
   Which shape am I?

   shape C rectangle

2. I have 4 sides.
   I have only 1 pair of parallel sides.
   Which shape am I?

   shape B Trapizoid

3. I have 4 sides.
   My opposite sides are parallel.
   I do not have any right angles.

   a. Which shape am I?

   shape A parallelogram

   b. Draw lines that divide my shape into a rectangle and two right triangles.

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Student A, part 2

4. Write three different clues for shape D.

D

Clue 1:

I have 4 right angles ✓. 1

Clue 2:

My opposite sides are parallel ✓ 1

Clue 3:

I have 4 sides ✓. 1

5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

They both have 4 sides ✓. 1

Write a statement that tells how they are different.

F is parallel and E is not ✓ 1

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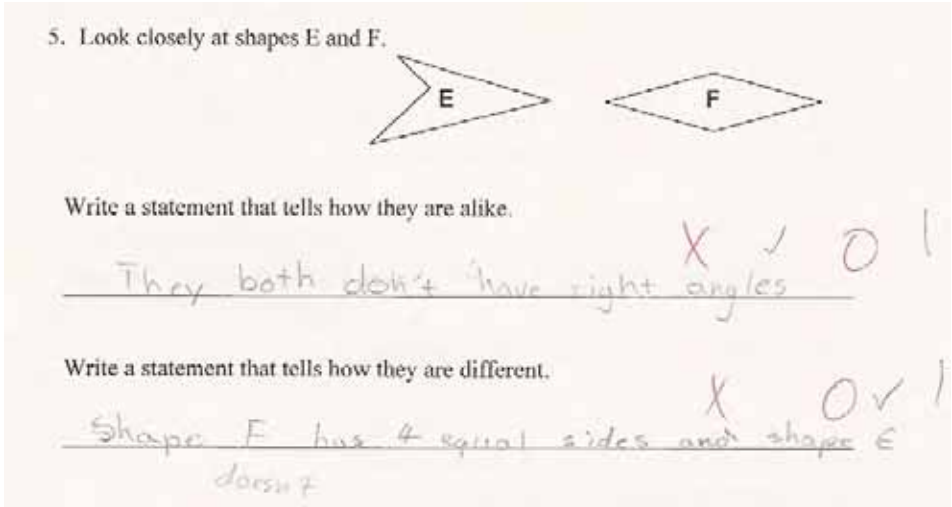
Recognizing the Seeds of Mathematical Ideas

Students sometimes come up with mathematical ideas not anticipated by the rubric or using imprecise language. *How do we, as teachers, develop the flexibility to see mathematical thinking in student work?* Look at the work of Student B. Notice that one scorer has given the student credit and one has not. *Has the student noticed important characteristics about the shapes?*

Student B



5. Look closely at shapes E and F.

E F

Write a statement that tells how they are alike.

They both don't have right angles

X ✓ 0 1

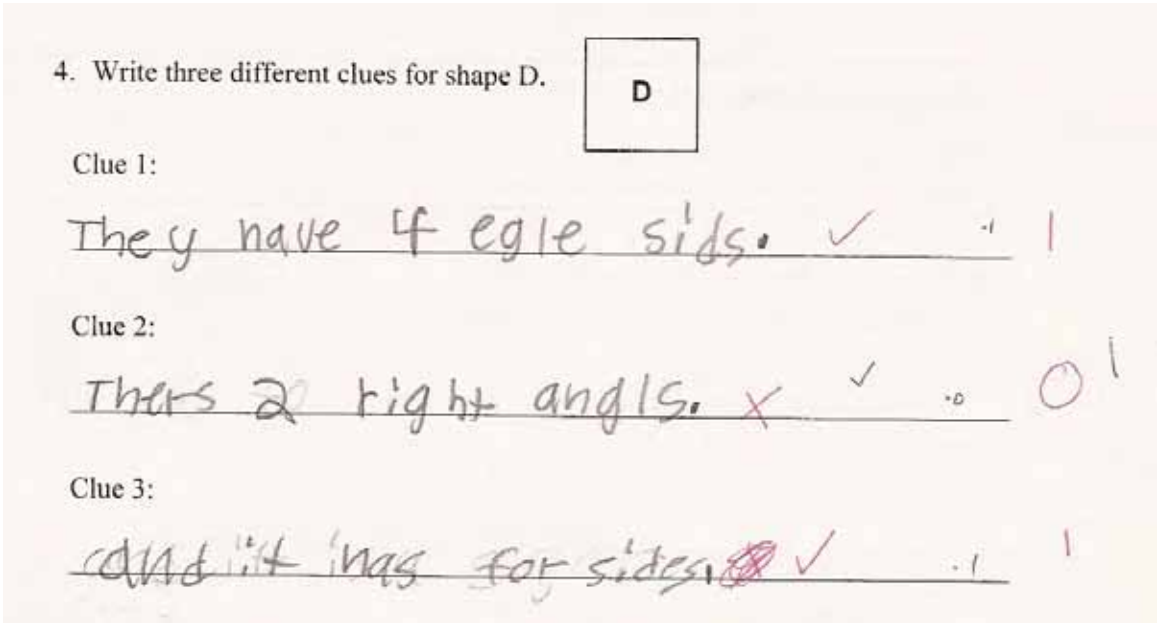
Write a statement that tells how they are different.

Shape F has 4 equal sides and shape E doesn't

X 0 ✓ 1

Look at the work of Student C. When taken alone, 2 right angles is not enough to describe shape D; neither is 4 sides or 4 equal sides. Clues need to be considered as a set. *Can you make a shape with 4 sides and 2 right angles that is not a square? Can you make a shape with 4 equal sides and 2 right angles that is not a square*?

Student C



4. Write three different clues for shape D.

D

Clue 1:

They have 4 egle sids. ✓ .1 1

Clue 2:

Thers 2 right angls. x ✓ .0 0 1

Clue 3:

and it has for sides ✓ .1 1

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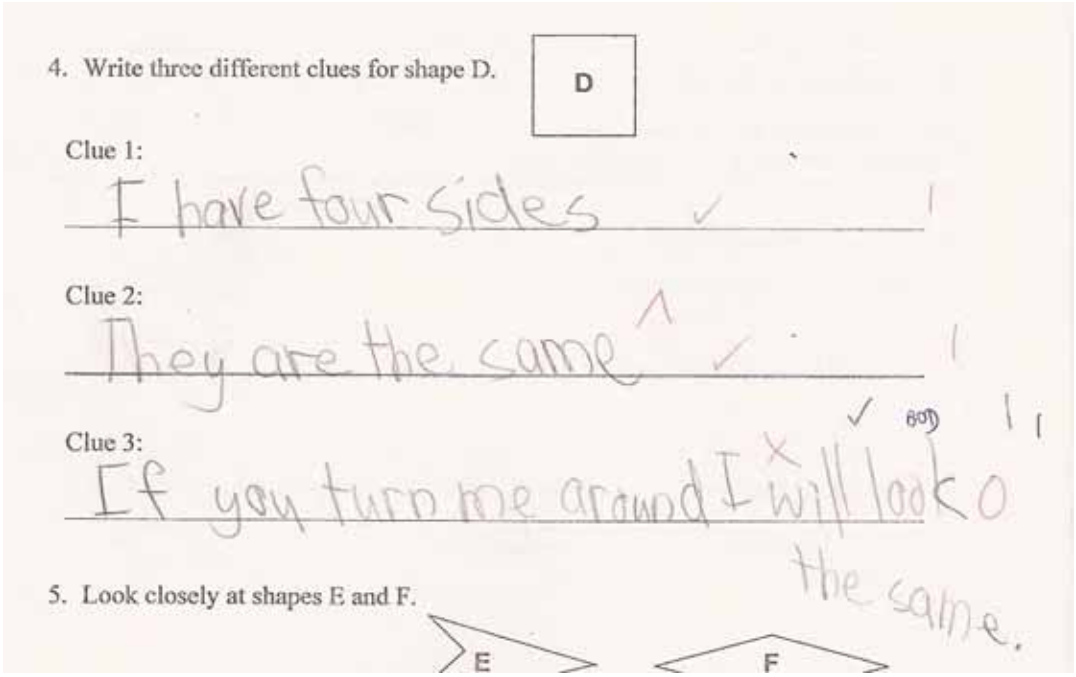
Student D has the seeds for the mathematical idea of rotational symmetry, but doesn't have the mathematical language to name the property. *How do we encourage students to think about important ideas, which might not be introduced formally until much later grades? How do we keep them looking for and discovering relationships?*

Student D

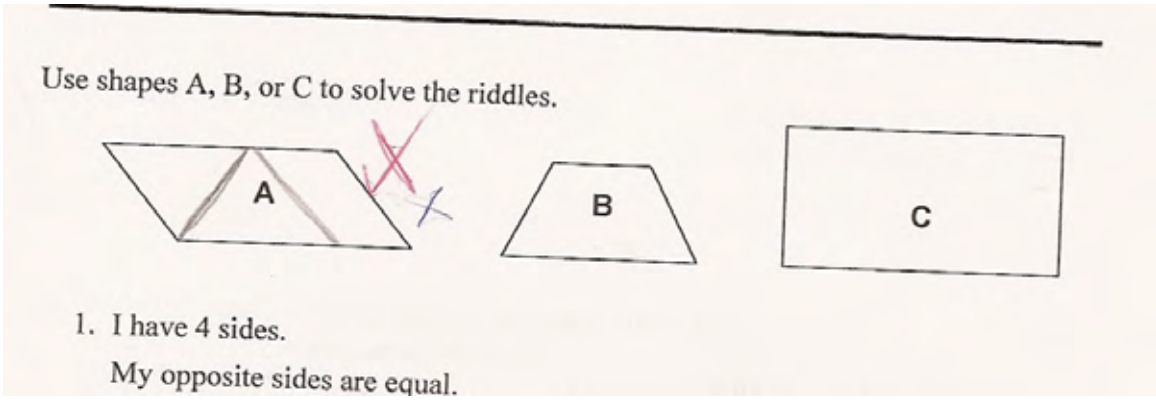


What's wrong with this picture?

Students had difficulty drawing in the lines to make a rectangle and two right triangles. *For each piece of student work, try to identify the attributes that they overlooked or incomplete definitions about a shape.*

Student E makes two triangles and a four-sided shape. *What is the student not understanding or focusing on?* Now look at part 4. The student comes up with for mathematical ideas about the square, but then uses none of those ideas in thinking about shapes E and F. *What might be a next instructional move for this child?*

Student E



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Shape E, part 3

Clue 1:

I have 4 sides. ✓ 1

Clue 2:

All my sides are the same. ✓ 1 3

Clue 3:

If you fold me, you get a symmetry (sp) ✓ 1

5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

They both have at least a pointed edge. ✓ 1

Write a statement that tells how they are different.

F looks like a diamond and E have a pointed edge that is pointed to the south. ✓

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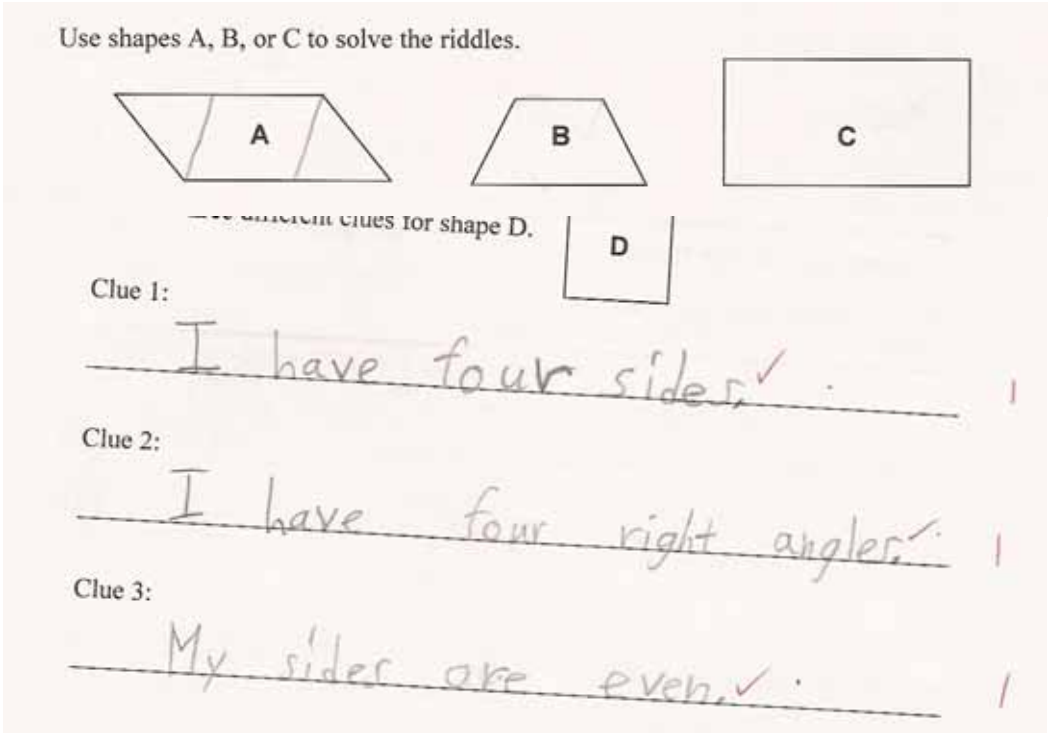
Student F has trouble with the drawing in part 3. What is the student forgetting? Examine the student's description of differences. The student is probably noticing the reflex angle in E, but does not know how to talk about it and leaves the description vague. We sometimes want students to have precise language. When encountering a new idea, students need to rely on everyday language and some mathematical language used in new ways. *What are some ways that we might hope third graders would describe this angle? What are the attributes from part 4 that might help the student in thinking about the differences between the two shapes? What could the student build on?*

Student F



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5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

They both have four sides. V.

Write a statement that tells how they are different.

The left side of E is different than F's left side. x

What are each of these students understanding and not understanding?

Student G

a. Which shape am I?

b. Draw lines that divide my shape into a rectangle and two right triangles.

Which Shape? Test 3

5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

They both have points in the same directions.

Write a statement that tells how they are different.

One is biger and one is smaller

Student H

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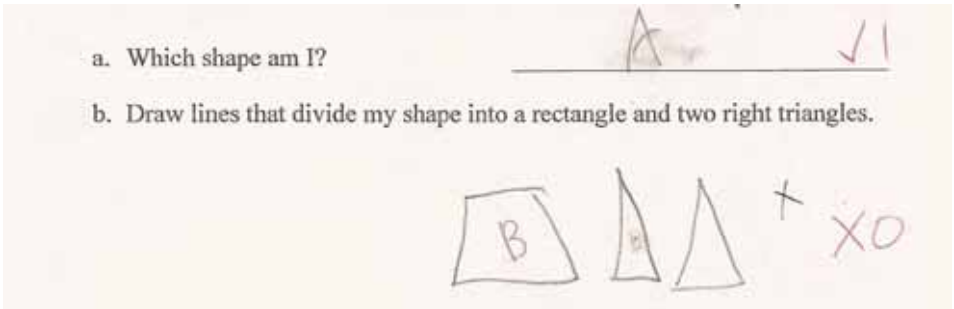
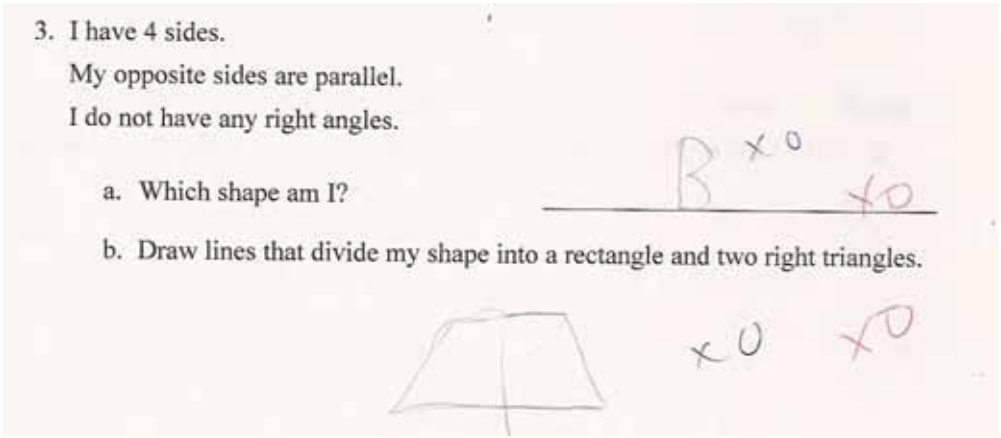


Do you think Student I is only asked to draw lines when looking for symmetry?

Student I



What are the important attributes of shape that we want students to pay attention to? How do we help them develop their sense of what to pay attention to?

Look at the work of Student J. *Does the student only think about angles as acute shapes?* Notice that the student in part 5 is thinking about the shape as a totality, rather than comparing parts. *How is this student thinking about the reflex angle?*

Student J

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Clue 1:

I have 4 sides ✓ 1

Clue 2:

I dont have any angles ✗ 0

Clue 3:

Each sides are the same ✓ length 1

5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

where the arow thats pointing ✗ 0 to the E if you pull it back it becomes the F

Write a statement that tells how they are different.

E has a triangle with no bottom ✗ 0 in it

Thinking about the angle in E is difficult for students. While we don't want students to memorize formal language about the angle, students should have opportunities to encounter and think about these types of big mathematical ideas and try to describe them. *What questions might you want to ask Student K to probe his thinking?*

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Student K

Clue 3:

the shap has four 90 angel.

5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

They both have a tip on the end.

Write a statement that tells how they are different.

shape e goes is and shap f goes out.

Look at the descriptions by Student L. *Would you have given them credit? Why are why not? Are the attributes accurate? Relevant?*

Student L

Clue 3:

you can draw a line of symetry between me and get 2 small rectangles 3

5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

they are both made out of triangles

Write a statement that tells how they are different.

shape E isn't equal and shape F is.

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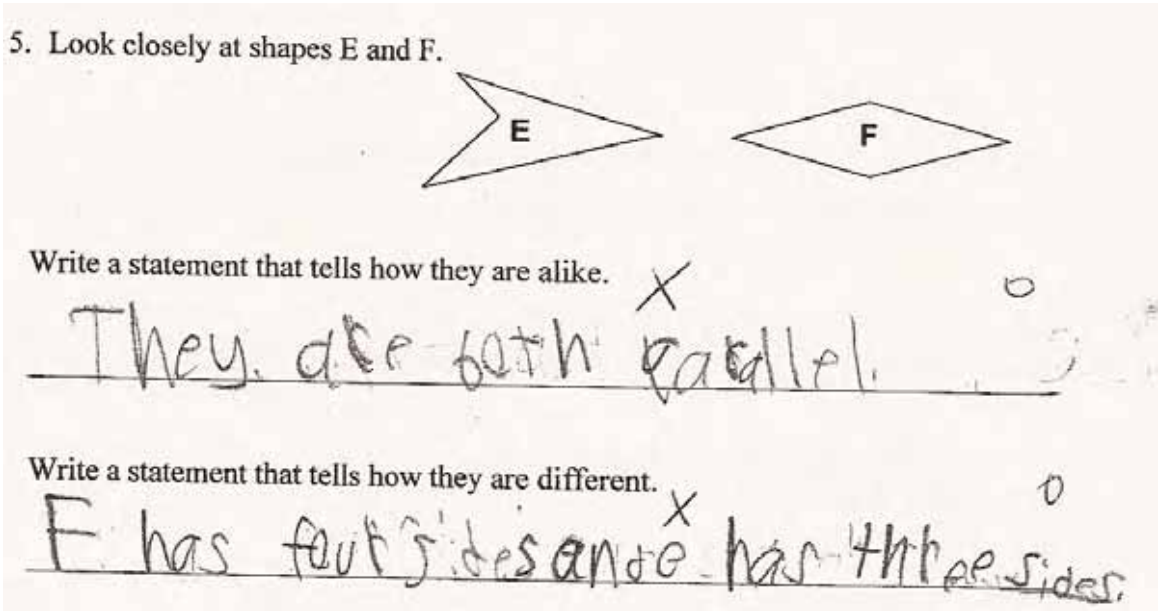
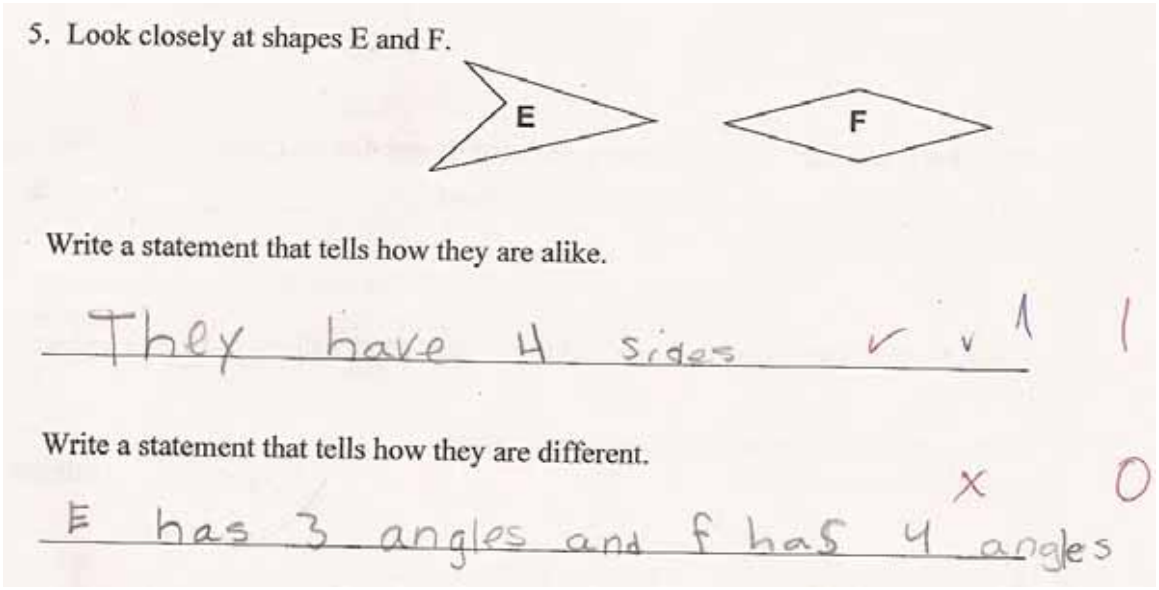
Some students struggled with just counting the sides and angles for the two shapes. See some typical responses by Students M and N.

Student M



Student N



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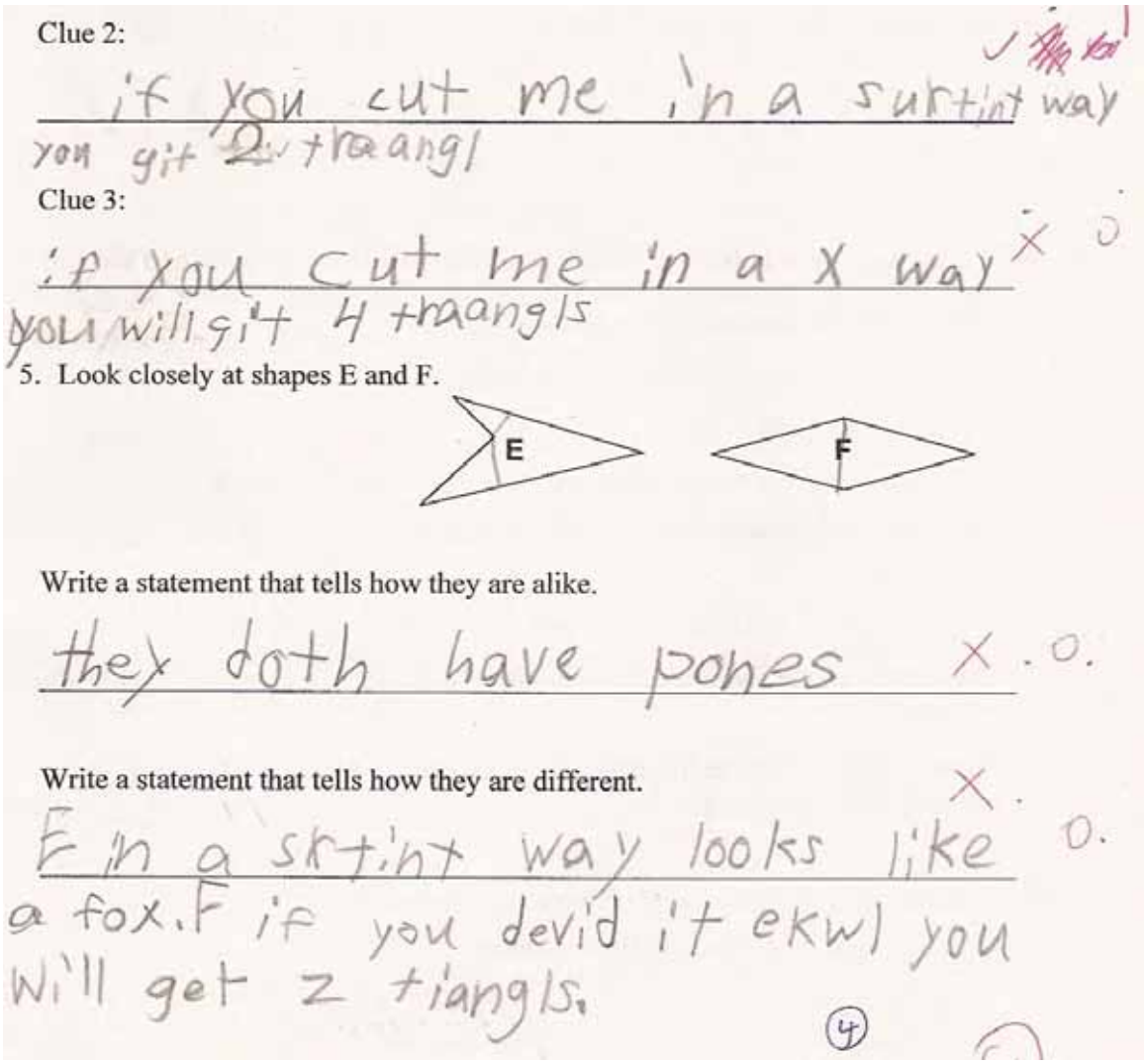
Finally some students, like Student O, are still operating at the lowest van Hiele level. They are thinking of what the shape looks like and how it can be subdivided.

Student O



Clue 2:

if you cut me in a surtint way you git 2 traangl

Clue 3:

if you cut me in a X way you will git 4 traangls

5. Look closely at shapes E and F.

Write a statement that tells how they are alike.

they doth have pones

Write a statement that tells how they are different.

E in a srtint way looks like a fox. F if you devid it ekwl you will get 2 tiangls.

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3rd Grade Task 4 Which Shape?

Student Task	Identify and describe shapes, using key attributes. Use clues to solve riddles. Identify how shapes are alike and different using geometric attributes.
Core Idea 4 Geometry and Measurement	Recognize and use characteristics, properties, and relationships of two-dimensional geometric shapes. • Identify and compare attributes of two-dimensional shapes and develop vocabulary to describe the attributes. • Use visualization, spatial reasoning, and geometric modeling to solve problems. • Recognize geometric ideas and relationships and apply them to problems.

Mathematics in this task:

- Understanding geometric attributes, such as angle size, number of angles, number of sides, parallel sides.
- Sorting and describing shapes using attributes.
- Comparing and contrasting shapes on geometric attributes.
- Composing and decomposing shapes.

Based on teacher observations, this is what third graders know and are able to do:

- Identify shapes by their attributes
- Describe attributes of a square, particularly 4 sides and 4 angles

Areas of difficulty for third graders:

- Drawing in lines to decompose a shape into a rectangle and two triangles
- Describing similarities and differences for E and F
- Correct vocabulary
    - Students used point or corner for angle
    - Closed shape, open shape
    - How to classify angles
    - They confused side and face and side and edge
    - Parallel sides

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MARS Test Task 4 Frequency Distribution and Bar Graph, Grade 3

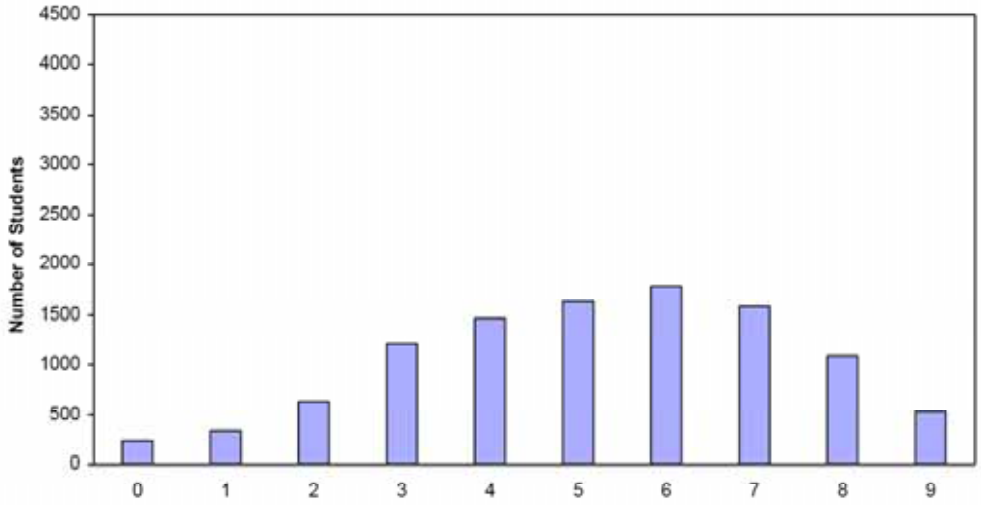
Task 4 - Which Shape?

Mean: 5.19 StdDev: 2.17

Table 18: Frequency Distribution of MARS Test Task 4, Grade 3

Task 4 Scores	Student Count	% at or below	% at or above
0	243	2.3%	100.0%
1	340	5.6%	97.7%
2	623	11.5%	94.4%
3	1209	23.0%	88.5%
4	1458	36.9%	77.0%
5	1630	52.5%	63.1%
6	1782	69.5%	47.5%
7	1583	84.6%	30.5%
8	1086	94.9%	15.4%
9	532	100.0%	5.1%

Figure 27: Bar Graph of MARS Test Task 4 Raw Scores, Grade 3



The maximum score available on this task is 9 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, 88%, could give 3 clues for describing a rectangle. 77% could also identify the shape with 4 right angles and opposite sides equal. Almost half the students, 47%, could identify all 3 shapes and give 3 clues for describing a rectangle. 30% could also draw in lines for subdividing a shape into a rectangle and two right triangles. 5% could meet all the demands of the task including comparing and contrasting shapes E and F, using geometric attributes. About 2% of the students scored no points on the task. Half of these students attempted the task.

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Which Shape?

Points	Understandings	Misunderstandings
0	Half the students with this score attempted the task.	They had difficulty describing attributes for a square. They gave information, such as: "no angles or no right angle, "looks like a box or piece of paper or a rectangle", or "its pointy".
3	Students could give 3 clues about a square.	They had difficulty matching a set of clues to a shape. For part 1 (4 sides, 4 right angles) 11% of the students picked the parallelogram.
4	Students could identify the rectangle from clues and give 3 clues for a square.	For part 2 (4 sides, only 1 pair of parallel sides) 23% picked the parallelogram and 11% picked the rectangle. For part 3 (4 sides, no right Angles, opposite sides parallel) 30% picked the trapezoid.
6	Students could identify rectangle, parallelogram, and trapezoid from their attributes and give attributes for a square.	30% of all students did not draw lines to subdivide a shape into a rectangle and 2 right angles. Students did not usually think carefully about angle size.
7	Students could identify shapes from attributes and describe attributes of shapes. Students could decompose a shape into rectangle and two right triangles.	Students had difficulty describing the similarities and differences between two shapes. 31% described the shapes as sharp or pointy.16% just noted that the shapes were different. 16% thought that E had only 3 angles or 3 sides. 7% tried to describe the concave angle with words like indent or goes in and goes out.
9	Students could identify shapes from attributes and describe attributes of a square. They could decompose shapes into component parts by attributes. They could also compare and contrast shapes based on geometric attributes.	

Implications for Instruction

Students should be able to think about the geometric attributes of shapes: number of sides, number of angles, types of angles, length of sides, and parallel sides. Students should be able to both identify a shape from its attributes and describe the attributes of a shape.

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Students need experiences with a rich variety of 2- and 3- dimensional shapes. Students should be pressed to describe how shapes are the same or different. This helps them become of aware of a variety of attributes and lays the foundation for understanding formal definitions at a later time.

Ideas for Action Research – The Logic of Classifying and Sorting

Look at the lesson below (from Teaching Student-Centered Mathematics, Grades K-3 by John Van de Walle). How might this lesson be modified to help students think about 3-dimensional shapes? Can you design your own set of shapes? Make sure you think carefully about the question posed by the Stop Sign.

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EXPANDED LESSON

(pages 221–222)

A complete lesson plan based on "Shape Sorts" can be found at the end of this chapter.

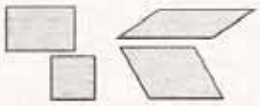
ACTIVITY 7.1

Shape Sorts

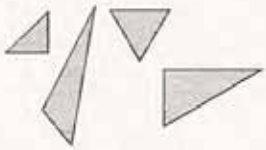
Have students work in groups of four with a set of 2-D Shapes similar to those in Figure 7.2. Here are several related activities that might be done in order:

- Each child randomly selects a shape. In turn, the students tell one or two things they find interesting about their shape. There are no right or wrong responses.
- Children each randomly select two shapes. The task is to find something that is alike about their two shapes and something that is different. (Have them select their shapes before they know the task.)
- The group selects one shape at random and places it in the center of the workspace. Their task is to find all other shapes that are like the target shape, but all according to the same rule. For example, if they say "This one is like our shape because it has a curved side and a straight side," then all other shapes that they put in the collection must have these properties. Challenge them to do a second sort with the same target shape but using a different property.
- Have students share their sorting rules with the class and show examples. All students then draw a new shape that will also fit in the group according to the same rule. They should write about their new shape and why it fits the rule.
- Do a "secret sort." You or one of the students creates a small collection of about five shapes that fit a secret rule. Leave others that belong in your group in the pile. The other students try to find additional pieces that belong to the set and/or guess the secret rule.

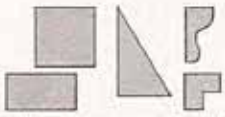
Shapes with curved edges



Opposite sides "go the same way"—parallelograms



Three sides—triangles



Shapes with a "square corner"—right angle

These all "dent in"—concave

FIGURE 7.3

By sorting shapes, students begin to recognize

Why do you think that the teacher should not say things such as, "Find all the pieces with straight sides," or "Find the triangles," and instead have students choose how to sort?

In any sorting activity, the students should decide how to sort, not the teacher. This allows the students to do the activity using ideas *they* own and understand. By listening to the kinds of attributes that they use in their sorting, you will be able to tell what properties they know and use and how they think about shapes. Figure 7.3 illustrates a few of the many possible ways a set might be sorted.

The secret sorting activity is one option for introducing a new property. For example, sort the shapes so that all have at least one right angle or "square corner." When students discover your rule, you have an opportunity to talk more about that property.

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